"Quaderni di Ricerca in Didattica (Matematica)", Supplemento n.4 al n. 19, 2009.
G.R.I.M. (Department of Mathematics, University of Palermo, Italy)

UNDERSTANDING THE TEACHER'S ROLE IN SUPPORTING STUDENTS' GENERALISATION WHEN INVESTIGATING SEQUENCES

Hélia Oliveira
Instituto de Educação
Universidade de Lisboa, Portugal
hmoliveira@fc.ul.pt

The aim of this paper is to analyse the teacher's role in supporting students' work in mathematical investigations, in particular, with one task which involves generalisation in sequences in the 8th grade, and the constraints the teacher faces when performing this role. The lessons were audio and video recorded and the teacher was interviewed in order to understand her options and perspectives about the classroom events. Five aspects of the teacher's role have being identified regarding her support to students' generalisation, namely, suggesting them to: display more steps in the sequence; draw a different representation, usually a table; establish a relation between the term and its position; look for familiar sequences in the numbers; and, use a proper variable. From the results it appears that to understand the teacher's role in this situation it is necessary to take into account: i) the nature of the other tasks that have been solved in previous lessons and its relation to this one; ii) the teacher's perspectives about what counts as generalisation and the different possible representations involved in the task; and, iii) finally, the compatibility of the goals established by the teacher for the task and for supporting students' work.

1. INTRODUCTION

This research develops in the context of one teaching experiment that promoted mathematical investigations in the classroom with 8th graders. The teacher's role is particularly demanding in classes where students develop this kind of activity, therefore it is important to understand the teachers' practice, and the challenges they face, in this context. In this paper I present the case of one teacher working in the classroom with one particular task that involves generalisation in the domain of algebra. Considering that the practice of algebra teaching still needs much more research (Kieran, 2007), two research questions are targeted in this paper: What are the characteristics of the teacher's role in supporting students' generalisation when investigating sequences? What constraints does the teacher face when performing this role and what are their origins?

2. THEORETICAL BACKGROUND

2.1. Teacher's role regarding mathematical investigations

The concept of mathematical investigation as an activity in the learning of school mathematics constitutes an educational metaphor: it intends to bring to the classroom some of the characteristics of the genuine mathematical activity. Students should develop mathematics in the sense that they have to formulate questions and conjectures, test them, build justifications and refutations, and to argue with the teacher and colleagues about their work. Teacher's role in promoting and nurturing this activity is a very relevant, complex and demanding one. It is necessary to support students on their work without leading them towards a desired solution, which implies, to pose good questions to students, evoke relevant information, exhibit mathematical reasoning, and promoting the reflection by the students (Ponte, Brocardo and Oliveira, 2003).

The interactions that take place in the classroom, according to Symbolic Interactionism, can not be reduced to a sequence of actions and reactions, because each participant controls his/her actions taking into account what he/she assumes as the others participants' understandings and expectations (Voigt, 1994). Even when participants do not explicitly present their points of view, the mathematical meaning is continuously negotiated. The mathematical discourse involved in mathematical investigations in the classroom assumes a particular nature, very different from the ones where the pattern is: the teacher formulates one question whose answer she/he knows, the student answers, and finally the teacher validates it (Wood, 1994). Interaction patterns and processes of communication are mutually constituted by the teacher and the students but it has to be the teacher who starts defining new frameworks for the activity when students are not acquainted with that.

2.2. Students' generalisations

Generalising is an intrinsic aspect of the mathematical activity, therefore its one of the highlighted processes in mathematical investigations in the classroom. As Steen states "mathematics is the science of patterns" (1988), and as such generalisation about patterns and regularities can be an important entry point into the process of generalisation. Research documents that pattern formulation can help students' introduction to algebra (Stacey and MacGregor, 2001). However, research also documents the students' difficulties when they start working with generalisation in sequences, because the transition from the particular to the general takes time (Kieran, 2007). For instance, it is very common that students use an additive approach to sequences, by connecting consecutive terms of the sequence, or fail in distinguishing between the Growing rule and the Position rule (Warren, 2006). Working with different representations can enhance students' skills in generalising, but research also shows that tabular representations of patterns may not help students to identify the "general relationships underlying patterns" (Kieran, 2007). Therefore the teacher needs to attend to these particularities of students' mathematical thinking and development in the context of their teaching of algebra.

3. METHOD

In this paper I will focus on one of the four mathematical tasks that the teacher proposed during a class experiment which intended students to develop an investigative approach to mathematical work. These lessons constituted the main context for the development of a research about the teacher's role in supporting students' generalisation. Accordingly, several mathematical tasks were presented to the teacher who chose those four that she considered more adequate to her 8th grade class, taking into account the mathematics curriculum and students' previous experiences. The questions on the task, its objectives and possible students' strategies were discussed previously with the teacher, in working sessions. The teacher always explored mathematically the tasks before the working sessions with the researcher and she had freedom to purpose alternative formulations of the task and to decide how to lead the lesson.

The class had 29 students, who were distributed among six groups, during these lessons. The work done in each of these tasks consisted of three moments: (i) a brief oral presentation about the task, by teacher, (ii) independent small group work, and (iii) whole class presentation and discussion of students' strategies. Due to limitations of space I simply analyse here the teacher's role regarding the first two moments.

The research methodology adopted in this study is qualitative and interpretative. The teacher's practices and her perspectives about her actions were analysed. I took part in the lessons as participant observer, and as I was introduced to students as one more teacher in the classroom. Sometimes I interacted with them to support their work. Data from the classroom were collected through video recording of the moments of whole class discussion and audio recording of the teacher's speech during the lesson. Additional data came from interviews, working sessions and post-lessons reflections with the teacher. All these data were transcribed and analysed by the researcher, and all the documents produced were read by the teacher.

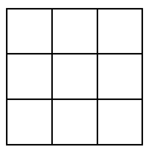
The teacher, Isabel, has about 20 years of experience, teaches in this school for about a dozen years and mainly in the secondary level (10th and 11st grades). She has a strong mathematical background, having studied applied mathematics for five years. Her love for mathematics and for teaching makes of her a teacher who derives great satisfaction from the profession.

4. RESULTS

4.1. Planning the lessons

Isabel regards this experiment as an opportunity for students and herself to develop new roles. She thinks that they will be more independent from her, trying to discover things by themselves, as they become "the 8th grade's little investigators", as she calls them. She views her role as someone who is there to support students' activity but will not direct them to a certain strategy or answer. The teacher prepares herself for these lessons solving the tasks and sometimes suggesting some modifications. In the case of this task, "Squares with matches" (fig. 1), she suggests to include the word "investigate" in the second question, because she fears that, otherwise, the students will only look for a number, like in the first question.

Squares with matches



How many matches are used in this square?
Investigate how many matches are necessary to build any square of this type.

Figure 1 – Squares with matches

The strategy she used to find the general expression was to construct a table, factorising the numbers on the sequence as n x (2n+2). However, she realizes that students can use very different strategies which can be very rich from a mathematical point of view. The figure in the task is seen by her as a possible motivation for students to engage in the activity. As they were acquainted with writing a general expression for a sequence (a topic worked four months ago but with easier sequences), the teacher plans to use two lessons for this task, including the discussion of the results, where the students present and their strategies and results.

4.2. Introduction to the task

In order to help the students to approach the task, in the beginning of the lesson, Isabel recalls with them that, some months ago, they have explored several numerical sequences starting from pictorial sequences. For instance, she mentions a task from the textbook with a sequence involving the areas of squares that was represented in a table, from which the students had to write its general expression. The teacher tells them that now they have a new challenge, because in this task only one term of the sequence is provided (the 3rd one): "you don't have the sequence: you will have to find it [by your own] and to build it". By recalling the students the general expression they wrote for that sequence, Isabel is implicitly showing that her intention for this "investigation" (this new sequence they have to "find") is that they obtain a generalisation, and to write it using symbols.

4.3. Teacher's support to students' activity

Right from the beginning of the lesson, Isabel realises that some groups do not understand, in the second question, what does it mean to construct "any square of this type". She helps them to identify the sequence of squares starting with the square which has one match on the side. The different groups tended to follow a recursive strategy, perhaps because in the previous tasks in this experiment they looked for a rule of construction relating the terms of the sequence (looking for patterns in the numbers, i.e. for properties of the numbers) and, as literature shows, this is easier for them. Since Isabel's intention is that they write down a general expression for the sequence, she gives the following suggestions to students:

To display more steps. In order to help them to recognize the law of construction of the sequence, the teacher sometimes suggests to students to go one or two steps further (n= 4 or 5). But, usually, that is not necessary since the most students take the initiative in doing it.

To draw a table (a different representation). This was suggested implicitly in the introduction done by the teacher. The usefulness of this suggestion is reinforced for Isabel since she observes that the first group that was able to find the general expression used this strategy. Then she decides to suggest it to several groups. For example, one group wrote different expressions for the sequence without success:

Paulo- Teacher, we can not find a sequence.
Teacher – You don't find a sequence?!
Miguel – No. What we have is 4, 12, 24, 40 and [these expressions] don't match these numbers.
(…)
Teacher – Ah! What you are saying is that you didn't find an expression, isn't it?
Paulo – Yes.
Teacher – So, let us see… Write down, as I told you before, [the numbers] in a table. This 4, is what? It is the numbers of matches that you used in a square with how many matches?
Ana – One.
Teacher – So, write: "side, 1" and here you put the total of matches that were used. How many?

However, this hint did not help most of the groups, since students need additionally to refocus their attention in two things: a "useful" factorization of the sequence terms and the relation between the term of the sequence and the respective order (instead of comparing consecutive terms). Sometimes this derives the students' attention from the strategy they were trying because they do not see its relation with the teacher's hint. What happens then is that most groups still look only to the numbers in the sequence and try to find a relation between consecutive terms. The teacher does not want to suggest the kind of factorization she used because she thinks that her role in this kind of tasks is only to support and not direct the students' explorations towards one process.

To establish a relation between the term and its position. When students are concentrated in finding a property in these numbers (for instance, some students conjecture that each number is the double of the previous one, because they notice that 24 appears in the sequence after 12), Isabel often tells them: "Try to relate these [the numbers in the sequence] with the number of matches on the side of the square". After a relative long period the teacher explicitly suggests to some groups to do some computation between the term and its position. This leads some groups to divide the numbers in the sequence by the respective order, obtaining the sequence of even numbers, but in some cases the students did not know what to do with these, because they could not find a relation between the terms and their respective order (like the double or the triple, as they expected).

To look for familiar sequences in the numbers. In the end of the first lesson, only one group succeeded in writing a general expression for the sequence, exactly using the same strategy as the teacher. Therefore, she decides to start the second lesson by remembering some of the familiar sequences they worked some months ago (natural numbers, even numbers and multiples of four) and recalling the meaning of "order" and "term" in a sequence. One student immediately establishes a relation between this hint and the work they have been doing in the previous lesson, recognizing the sequence of even numbers starting in four. Interestingly, the students in one of the groups, even after having factorized the terms as the product of the order by the sequence of the even numbers starting in four, do not identify it as that sequence and still focus their attention on consecutive terms. The teacher's questioning here becomes crucial:

Teacher- Haven't you find an expression?
Daniel – I found something but still we had to come back and then I thought that is useless because…
Teacher – You always had to get to the previous term, is it?
Daniel – To get the number of matches, we multiply by four and then we multiply by plus two, by six, then by eight, by ten and so on… but it doesn't do…

The teacher now asks them to write down the products Daniel mentioned, and one of the girls in the group refers that those are even numbers. Relying on this comment the teacher helps them to recognize the two familiar sequences in that product.

To use a certain letter for the variable. In some groups the teacher mentions explicitly the need to choose a letter for variable in the general expression of the sequence. She recalls them that *n* can be an appropriate letter to use in the expression, since the order is a natural number. Isabel makes this suggestion, because she thinks this can help the students to "remember the work they have done before with the sequences".

4.4. Teacher's difficulties in supporting students' generalisation

The goal established by the teacher for this task is the generalisation of the sequence presented using symbols, as I mentioned before. However, her role in supporting students' activity in generalising did not succeed as expected in some instances. Isabel recognizes it and expresses some frustration because some students did not manage to find the general expression for the sequence:

> "I became a bit frustrated in this lesson. And that comes from… that they realised [the law of construction of the sequence], they had everything done, but it is that bridge … to the generalisation that they could not go through. And I asked myself: *How am I going to explain this to the students without being directive?*"

In this section, I describe in detail some of the difficulties the teacher faced in supporting students' generalising activities, organized by topics.

Recursive approach. When approaching the groups that are trying to write down a general expression for the sequence, the teacher usually suggests them to establish a relation between the term and its position. However, sometimes she focus their attention in a possible relation between consecutive terms, as it happens, in the following episode, with a group that determined the terms of the sequence until n=5, but do not seem able to grasp the general expression:

Teacher- Now you have to look at the "behaviour" of the sequence, one that from four goes into 12, from 12 goes to 24, from 24 to 40, from 40 to 60…
Ana – It is always the double!
Fábio – No.
Teacher – The double of what?
Beatriz – No, it is not.
Teacher – Look at this "behaviour". There is always something there that is common. There is always something that we do when we pass from…
Fábio – (interrupting) Four times three, 12.
Beatriz – They're always multiples of four.

What the teacher intends with this support is that they notice what is happening to the terms in a functional way, but the students do not interpret it in that way. They just look for patterns in the numbers in the sequence, trying to find proprieties common to all of them.

Law of construction. At the same time, the teacher seems not to realize, in the class, that she uses different terms when she refers to the intended outcome of the students' activity within this task. Sometimes she asks, simultaneously, for the law of construction of the sequence and for the general expression, apparently referring herself to the same entity.

The use of symbols in recursive reasoning or in a rule. Some students identify the sequence in a general but recursive way, describing the rule. For instance the students in one group state, "We sum the difference between the two previous ones and add four to obtain the following", and after a period of work present "$n + (n - n_1) + 4$", meaning $u_{n+1} = u_n + (u_n - u_{n-1}) + 4$, with n natural. The teacher verifies the meaning of the recursive rule they present but does not discuss with them their use of the letters. A similar situation happens with one group that notices that the difference between consecutive terms is the sequence $4n+4$. However, the teacher does not help them to write down the recursive rule, even without symbols, again, because she is much focused that they obtain the general expression.

The use of the figure. The majority of the groups only look at the figure in the initial phase to count the number of matches in an organized manner. After that they centred their exploration on the numbers they obtained. The teacher does not suggest students to focus on the way they counted the number of matches to obtain the terms of the sequence. One student arrived at the expression “l x (l-2+l) + (lx4)” (l, the number of matches on the side of the square) counting the matches in the perimeter of the square and the matches inside of it, but the teacher had no intervention on this.

Different factorizations. The teacher is not able to help for the generalisation those students who factorized the numbers in the sequence in a different way from herself. For instance, one group gets the sequence: 4x1, 4x3, 4x6, 4x10… but they can not write down the general algebraic expression. The teacher observes the factorization they wrote and tells them to look closely to what they have done in order to get a general expression. Nevertheless, she abandons the group without a suggestion so that can they obtain the general expression, because she did not anticipate that solution when she explored the task by herself.

Globally, the teacher feels that she did not help too much the groups that were able to find the general expression: “I was conscious that if I were to say something I would say everything”. She recalls that usually she only suggested the groups to relate the numbers in the sequence, and in the second lesson that they focus in the number of matches in the side of the square. Otherwise, she felt that she could be putting in danger the nature of the task: an investigative one.

Isabel also explains that she prevented herself from giving hints to some students, when there were facing difficulties in writing the general expression, because she did not want to influence their own strategies: “As you know, there are several paths and I was afraid that by giving them hints they would go in the same direction that I have followed, and that might not be the easiest [path] for them”. Therefore she thought that she might not be giving the best support to students.

5. DISCUSSION AND CONCLUSIONS

The purpose of this paper is to analyse the teacher’s role in supporting students’ generalisation when they are investigating numeric sequences, as well as to identify the constraints on this role and their origins. The teacher’s role is intimately linked to the students’ interpretation and approach to the task. Students’ first approach to this task is to compare consecutive terms of the sequence, and some of them explain the relation through a general rule. They reveal great difficulty to depart from this kind of reasoning to the one that expresses the functional relation. When the teacher recalls explicitly some of the sequences of special numbers they worked some time ago, some groups succeed to write the general expression (sometimes with some direct help from the teacher). In this section I summarise and discuss these issues.

5.1. The teacher’s role in supporting students to generalise

In the previous section I identified five aspects of this teacher’s role in supporting the students’ activity, namely, suggesting them to: *display more steps in the sequence*; *draw a different representation, usually a table; establish a relation between the term and its position; look for familiar sequences in the numbers; and, use a proper letter for the variable*. These can be seen as having mainly the influence of the:

(i) strategy the teacher employed to solve the task when she planned the lesson;

(ii) teacher’s expectations about the students’ knowledge on sequences of numbers;

(iii) intended outcome, a specific understanding about generalising as presenting a general symbolic expression.

It becomes clear from here that the teacher's support to students in this task has been mainly focused on one specific route to generalisation. Probably because her main experience has been in the teaching of mathematics to secondary grades, she did not reflect substantially about how to support middle grade's students to go from the particular to the general, and how to express a functional relation. These are very important and difficult tasks for teachers as the vast literature on the learning of algebra shows (Kieran, 2007).

5.2. Some constraints to the teacher's role

The intended support for students' activity sometimes did not succeed as expected. This fact may be linked to several aspects, which I will discuss, in detail, in this section:

The nature of the previous tasks, based upon patterns and regularities (mainly in terms of numbers' properties), created a false expectation in students that when they operate on the numbers of the sequence somehow they will come to that kind of regularity. The teacher, apparently, did not consider this influence on students, and in some instances she still reinforced the perspective of looking to the sequence as a list of numbers with a particular property, because she did not realised that this would not help them to get at the general expression as she intended to.

The generalisation involved in this task is much difficult than that of the previous ones, when students worked with familiar sequences of numbers or in this teaching experiment, where students just had to identify regularities and to work with sequences that were much simpler. The teacher did not take this into account when she prepared herself for this lesson. Even when she tries to push the students to the strategy she used, she misses the fact that it is more complicated to reason upon the product of two sequences than of only one.

The intended goal for the task and the role the teacher wants to perform conflict with each other. The teacher's role is informed by the meaning that she attributes to the task's nature. Being integrated in a teaching experiment with mathematical investigations, the teacher's perspective is that she should not be directive in her support to the groups. However, this task was transformed by the teacher into a problem of finding the general expression for one sequence. In here, the teacher faces the dilemma of putting the emphasis on the kind of activity that she intended students to develop (an investigative one) or on the activity's outcome. This situation is particularly evident in the case of those students who factorise the sequence in a different way from the teacher but do not receive any help or suggestion from her to get to the generalisation. Therefore this internal contradiction in what the teacher sees as the main goal for the students' activity, leads her to perform a non consistent role during the students' work on the task.

The teacher's notion of what constitutes a desirable generalisation in this context became problematic for her role. For this teacher the work with sequences in mathematics, in the 8^{th} grade, serves as an introduction to the topic "operations with polynomials". Therefore the functional dimension is less focused by the teacher who, as mentioned above, centres on the identification of known sequences of numbers to formulate the general symbolic expression. When the teacher suggests the use of variables (for instance, *n* for the natural numbers) these appear more as letters that represent particular values than relationships (Becker and Rivera, 2005). On the other hand taking into consideration that these are middle school students, it could be more appropriate to give them the opportunity to formulate in words or symbols the rules they found, something that they did not experienced before, and support them to be rigorous in that. The teacher's experience with the secondary school mathematics, sustained by a sophisticated mathematical knowledge, may have an influence on her perspective about what is a proper generalisation according to the objectives of the mathematics syllabus. This perspective stands in line with the results of the research by Bergqvist (2005) where some secondary teachers showed their preference for algebraic approaches and considered that the use of advanced mathematical expressions by the students was an evidence of high performance.

The role of different representations in supporting students' trajectory to generalisation is not completely taking into attention by the teacher in these lessons. Students could have formulated the generalisation from the visual representation of the sequence, as the teacher noticed when she selected the task. However as she did not what to push them into a particular strategy she did not suggest them to concentrate in that representation. Research shows that students who use the visual representations of sequences tend to employ strategies with a focus in the relationships (Becker and Rivera, 2005) however, secondary teachers have not been much acquainted with this idea.

6. CONCLUDING REMARKS

The goal of this paper was not to evaluate, in a simplistic way, what the teacher did or did not do well when she tried to perform a different role in supporting the students' mathematical activity. This research, centred on the mathematics classroom practice, allows us to understand the complexity of the teacher's role when students work on tasks that involve generalising and induction processes. This is particularly important in the context of the reformed Portuguese mathematics' syllabus that is being implemented in elementary and middle grade schools, which attributes great importance to generalisation in the context of algebra. This will, certainly, constitute an important challenge for many teachers. The preparation of these lessons will demand from teachers to think about appropriate tasks and approaches that might help the students to progress in this process. For instance, it is important for students to see the teacher generalising and to understand that he/she values their "attempts at generalisation" (Mason, Drury and Bills, 2007, p. 56).

According to the review of literature focusing on learning and teaching of algebra made by Kieran (2007), the practice of algebra teaching is one of the major areas that is still beginning to be studied but, like in other areas of teaching, it is clear that the teachers' content knowledge on algebra does not assure the pedagogical content knowledge necessary to teach this theme. It is necessary to have the knowledge, informed by research and by personal practice, of how students' subject-matter specific knowledge develops. In the case of the present study, the analysis of the teacher's role has been informed by the research and knowledge developed about the learning of algebra. Therefore, I fully agree with the recommendation that studies should more often attend to both the teaching and learning of algebra and their relation (Kieran, 2007). New understandings about the practice of algebra teaching can further be developed if the research on the mathematics classroom centres on the teacher's practice itself, and at the same time on the students' learning and thinking processes.

In this study I tried to understand the teacher's options and difficulties she faced taking into account her perspectives and the context where teaching occurs. The view about teaching, and particularly about good teaching, of those that are mainly outside school practice is often very different from the teachers' view (Wilson, Cooney and Stinson, 2005) therefore it is important to consider the teacher's voice when developing research in the classroom.

The studies on the teacher's practice also contribute to question teacher education and to improve it. The teacher in this study experienced a dilemma in these lessons: How to put in practice the new curriculum recommendations for problem-solving or investigative approaches to mathematical learning and, simultaneously, lead the students to learn specific topics, some of them much challenging as algebra? Difficult issues such as this need much attention from teacher education but do not have easy solutions. Developing pedagogical content knowledge about algebra and reflecting on this kind of experiences with (significant) others may help teachers to develop the necessary expertise to perform their roles within these demanding classroom scenarios. Teacher education faces the challenge of providing these significant others in relevant contexts of practice.

RERERENCES

Becker, J. R., & Rivera, F. (2005). Generalization strategies of beginning high school algebra students. In H. L. Chick & J. L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 121-128). Melbourne, Australia.

Bergqvist, T. (2005). How students verify conjectures: Teachers’ expectations. *Journal of Mathematics Teacher Education, 8*, 171–191.

Kieran, C. (2007). Learning and teaching algebra in the middle school through college levels: Building meaning for symbols and their manipulation. In F. K. Lester, Jr. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 707-761). Charlotte, NC: NCTM-IAP.

Ponte, J. P., Brocardo, J., & Oliveira, H. (2003). *Investigações matemáticas na sala de aula*. Belo Horizonte, Brazil: Autêntica.

Mason, J., Drury, H., & Bills, L. (2007). Studies in the Zone of Proximal Awareness. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential Research, Essential Practice* (Vol. I, pp. 42-58). Australia: MERGA.

Stacey, K., & MacGregor, M. (2001). Curriculum reform and approaches to algebra. In R. Sutherland, T. Rojano, A. Bell & R. Lins (Eds.), *Perspectives on school algebra* (pp. 141-154). Dordrecht, The Netherlands: Kluwer.

Steen, L. (1988). The science of patterns. *Science, 240*, 611-616.

Voigt, J. (1994). Negotiation of mathematical meaning and learning mathematics. *Educational Studies in Mathematics, 26*, 275-298.

Warren, E. (2006). Teacher actions that assist young students write generalizations in words and in symbols. In J. Novotná, H. Moroavá, M. Krátká & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 377-384). Prague, CZ.

Wilson, P., Cooney, T., & Stinson, D. (2005). What constitutes good mathematics teaching and how it develops: Nine high school teachers’ perspectives. *Journal of Mathematics Teacher Education, 8*, 83-111.

Wood, T. (1994). Patterns of interaction and the culture of mathematics classrooms. In S. Lerman (Ed.), *Cultural perspectives on the mathematics classroom* (pp. 149-168). Dordrecht, The Netherlands: Kluwer.